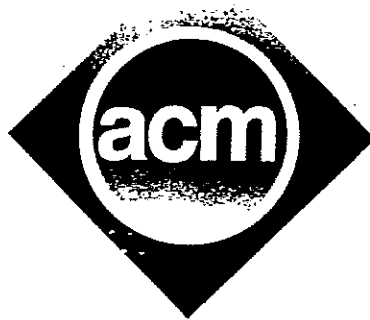


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RECONSTRUCTION OF POSSIBILISTIC SYSTEMS WITH INCOMPLETE INFORMATION †

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ABSTRACT

The reconstruction principle of inductive inference is based on reconstructability theory and offers an unorthodox approach to inductive reasoning, which has applications in diverse fields such as pattern classification, expert systems, generalized rule induction and distributed sensor integration. Reconstructability theory emphasizes the relationship between parts and wholes, the relationships between systems and subsystems, and more specifically, the relationship between states and substates. The two problems in reconstructability theory are referred to as the reconstructability problem and the identification problem. In this paper, we consider reconstruction and identification of possibilistic systems when information is not available in its entirety. We introduce the concept of the partial reconstruction hypothesis, and compute the unbiased reconstruction and the reconstruction families implied solely by the partial information. A possibilistic version of the probabilistic algorithm is proposed to determine the unbiased reconstruction, and the reconstruction families have been identified by transforming the possibilistic system constraints to max-min fuzzy relation equations.

Key Words: Possibilistic system, Fuzzy Relation Equations, Reconstructability Problem, Identification Problem, Reconstruc-

tion Family, Reconstruction Hypothesis, Structure System, Reconstruction Principle of Inductive Inference.

1. Introduction

The reconstruction principle of inductive inference is based on reconstructability theory and offers an unconventional approach to inductive reasoning, which has been applied to diverse fields such as pattern classification, expert systems, generalized rule induction and distributed sensor integration [22, 25, 37]. Reconstructability theory emphasizes the relationship between parts and wholes, the relationships between systems and subsystems, and more specifically, the relationship between states and substates. Reconstructability theory relates to two types of problems, namely, the reconstruction problem and the identification problem. The former deals with the process of reconstructing a given system under a given criterion from the knowledge of its subsystems and, during this process, identifying those subsystems that are instrumental in the reconstruction. The latter allows the identification of an unknown system from the knowledge of its subsystems. The solution procedures associated with these two problems are referred to as Reconstructability Analysis, abbreviated as RA. Origins of RA can be traced to Ashby's work on constraint analysis in the early sixties [1, 27], though a formal framework of RA did not exist until the late seventies [2, 23-24] and early eighties [3-6]. Since the advent of reconstructability theory, significant efforts have been directed towards research in this area, resulting in the emergence of a variety of new algorithms and applications. Solution procedures aimed at these two problems have been developed and implemented [9-10, 13-22, 25-28, 30-31, 38]. In this section, we shall

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introduce the basic terminology and the preliminary concepts in reconstructability theory [4, 18] and describe the motivation for our research.

1.1 Systems and States

Intuitively, a system is simply a data set which consists of the tuples of the form $\langle v_1, v_2, \dots, v_n, f \rangle$, where v_1, v_2, \dots, v_n are variables or attributes and f is a function defined over these variables. This function may be a probability distribution function, a possibilistic behavior function, a selection function, a fuzzy set membership function or any arbitrary or non-linear function. A state in a system is simply a combination of the attribute values in a given order. Formally, a system is defined as a six-tuple

$$B = (V, W, s, A, Q, f) \quad (1.1)$$

where $V = \{v_i | i \in 1, 2, \dots, n\}$ is a set of variables; $W = \{v_j | j \in 1, 2, \dots, m, m \leq n\}$ is a family of state sets; $s: V \rightarrow W$ is an onto mapping which assigns to each variable in V , one state set from W ; $A = s(v_1) \times s(v_2) \times \dots \times s(v_n)$ is the set of all potential aggregate states; Q is a set of real numbers; and $f: A \rightarrow Q$ is a system function which represents the information regarding the aggregate states of the system.

1.2 Subsystems and Substates

A subsystem is a data set whose variables form a proper subset of the variables of the system and a function g is defined over variables in the subset. It consists of the tuples of the form $\langle u_1, u_2, \dots, u_m, g \rangle$, where $\{u_1, u_2, \dots, u_m\} \subset \{v_1, v_2, \dots, v_n\}$. A substate in a subsystem is a combination of attribute values present in that subsystem in a given order. The concept of a system and subsystem is a relative one. A system can be regarded as a subsystem of a larger system and a subsystem can be regarded as a system (i.e., a supersystem) of a smaller system. Formally, given a system as defined above, a collection of a total of q subsystems, together called a structure system or a reconstruction hypothesis, is defined as

$$S = \{^k B\} = \{(^k V, ^k W, ^k s, ^k A, ^k Q, ^k f) | k \in \{1, 2, \dots, q\}\} \quad (1.2)$$

if and only if, for each k , following condition are satisfied:

- (1) $^k V \subset V$,
- (2) $^k W \subseteq W$ such that $^k s$ is onto,
- (3) $^k s: ^k V \rightarrow ^k W$ such that $^k s(v_i) = s(v_i)$ for each $v_i \in ^k V$,
- (4) $^k A = \times_{v_i \in ^k V} ^k s(v_i)$,
- (5) $^k Q = Q$,
- (6) $^k f = [f \downarrow ^k V]$.

Elements of set S are referred to as subsystems of system B . $[f \downarrow ^k V]$ is called the projection of f on $^k V$, which considers only the variables in $^k V$. Essentially, $[f \downarrow ^k V]$ is a mapping from substates in $^k A$ to Q , that is,

$$[f \downarrow ^k V]: \times_{v_i \in ^k V} s(v_i) \rightarrow Q \quad (1.3a)$$

such that

$$[f \downarrow ^k V](\beta) = g([f(\alpha) | \alpha > \beta]), \quad (1.3b)$$

where $\alpha > \beta$ means β is a substate of α (or α is a superstate of β), and g is determined by the nature of function f . Note that it is not always possible to derive subsystems using projection functions in the real world where different subsystems may be observed by different teams of observers or by using different experiments. This gives rise to the issues of local and global inconsistencies in the data.

1.3 Reconstructability Problem

Let B be a behavior system defined by (1.1). Let S be a structure system defined by (1.2). S is said to be a meaningful reconstruction hypothesis of B if and only if it contains the subsystems of B such that

$$\bigcup_{k \in N_q} ^k V = V, \text{ and} \quad (1.4)$$

$$(\text{for all } j, k \in N_q) (^j V \subseteq ^k V \Rightarrow j = k). \quad (1.5)$$

Condition (1.4) is called the *covering condition* and guarantees that all variables of B are included in S . This means that the reconstruction of B from S is logically possible. Condition (1.5) is called the *irredundancy condition* and ensures that S contains no redundant information.

1.4 Identification Problem

Let S be a structure system defined by (1.2). Then S is said to be a reconstruction hypothesis (hypothetical representation) of an unknown overall system B provided the following six conditions hold true:

- (1) $V = \bigcup_{k \in N_q} {}^k V$,
- (2) $W = \bigcup_{k \in N_q} {}^k W$,
- (3) $s: V \rightarrow W$ such that $s(v_i) = {}^k s(v_i)$ for each $k \in N_q$,
- (4) $A = \times_{v_i \in V} s(v_i)$,
- (5) $Q = {}^k Q$ for each $k \in N_q$,
- (6) $f: A \rightarrow Q$ such that $[f \downarrow {}^k V] = {}^k f$ for each $k \in N_q$.

It is obvious that B , which is unknown, should be compatible with S . Potentially, there will be more than one overall systems compatible with S . The set of all these systems is called *reconstruction family* of S , denoted by B_S . Let F_S be the set of all system functions corresponding to the overall systems in B_S . Since the elements in F_S and B_S are in one-to-one correspondence, and the elements of B_S differ only in system functions, B_S and F_S can interchangeably be referred to as reconstruction family.

As discussed previously, if the behavior functions $\{{}^k f\}$ of a reconstruction hypothesis are projections of an overall behavior function f , then the reconstruction hypothesis is *consistent*. A reconstruction hypothesis is said to be locally consistent if the following condition, called *local consistency condition*, is satisfied:

$$(\text{for all } j, k \in N_q)$$

$$([{}^j V \downarrow {}^j V \cap {}^k V] = [{}^k V \downarrow {}^j V \cap {}^k V]). \quad (1.6)$$

A reconstruction hypothesis S is said to be *globally consistent* if the reconstruction family of S is non-empty. It is usually the case that the reconstruction hypotheses satisfy local consistency condition (1.6) as well as the covering condition (1.4) and the irredundancy condition (1.5).

1.5 Advances in Reconstructability Analysis

The discipline of reconstructability analysis is well developed for probabilistic behavior functions. Of greater significance has been Jones' work [13-21]. Motivated by Lewis' study [29] on approximation of probability distributions to reduce storage requirements, Jones [13-15] provided the alternative methods of solution using minimal and limited information. He introduced the concept of null extension and k -system theory, and extended the previous results for the case when system functions were allowed to be any non-linear function, and this was accomplished only by means of the limited independent information [13-18]. He further generalized these results to hold good for incomplete and arbitrary data [19]. The concept of null extension and the advent of k -system theory has greatly extended the realm of RA methodology. By transforming any non-linear system to a dimensionless system, it is possible for RA to cover most non-linear functions. Using the concept of null extension, it is possible to divide the whole state space into disjoint equivalent classes and to proceed further by just picking only one state from each class. We use the similar concept to study possibilistic systems with incomplete information.

The principle of maximum uncertainty is used to compute the unbiased reconstruction from possibilistic structure systems. Cavallo and Klir [6] considered the reconstruction of possibilistic behavior systems. Using the principle of maximum ambiguity, which was later revised to the principle of U-uncertainty, they introduced methods for computing the unbiased reconstruction and reconstruction families of possibilistic structure systems. They proved that a possibilistic join procedure, the one similar to the probabilistic one, computes the unbiased reconstruction and that there is no need to employ an iterative procedure for the structure systems with loops. Higashi, et. al. [10] demonstrated that the reconstruction family of a given structure system is equivalent to the set of solutions of a special kind of fuzzy relation equations. The partially ordered solution set contains the minimal solutions and the unique maximum solution. Identifying these maximum and minimal elements only suffices to determine the whole reconstruction family.

1.6 Motivation

In real life, there may be situations when information is not available in its entirety, or it may be cost prohibitive to observe all the states in an experiment. For example, in genetics, where there are dominant and recessive genes, some states are readily observed whereas the knowledge of others requires expensive testing. Therefore, the task of working with limited information is of paramount concern in these circumstances. It is the objective of this paper to study such problems. In this paper, we first present a method for determining the unbiased reconstruction for possibilistic behavior functions using partial information. Then we describe an algorithm to determine the reconstruction family of possibilistic systems. As noted before, the problem of computing reconstruction family of possibilistic systems can be translated into the problem of solving a set of a special kind of fuzzy relation equations [10]. We use this equivalence in solving the identification problem.

2. Measures of Uncertainty for Possibilistic Systems and Unbiased Reconstruction

Let $B = (V, W, s, A, Q, f)$ be a possibilistic behavior function as defined in (1.1). Let $S = ({}^k B)$ be the reconstruction hypothesis as defined in (1.2). All the symbols have the same meaning as defined earlier except that the functions f and $({}^k f)$ are possibilistic behavior functions. Similar to the probabilistic systems, the possibilistic systems also satisfy covering condition, irredundancy condition and local consistency condition as defined in (1.4), (1.5) and (1.6) respectively. Let $Z \subset V$, then $[f \downarrow Z]$ is the projection of a possibilistic function in that it involves only those variables which are in set Z . Formally, $[f \downarrow Z]$ is defined as

$$[f \downarrow Z]: \prod_{v_i \in Z} s(v_i) \rightarrow [0, 1] \quad (2.1a)$$

such that

$$[f \downarrow Z] = \max_{\alpha \in \beta} f(\alpha). \quad (2.1b)$$

A justification of this definition has been provided in [39-41].

Evaluation of the reconstruction hypotheses for possibilistic systems relates to either of

the following two problems, depending whether or not the overall system is known. These two problems are the reconstructability problem and the identification problem. The identification problem requires the computation of the reconstruction family of B denoted by B_S (or F_S). To choose a single system function f_S from F_S requires some additional assumptions, depending on whether some extra information about the overall system under investigation is available or not. In the event not having such information, f_S should be maximally non-committal except for the following condition:

$$[f_S \downarrow {}^k V] = {}^k f \text{ for all } k \in N_q. \quad (2)$$

For a probability distribution, this amounts to saying that the set $\{f_S(\alpha) \mid \alpha \in \Omega\}$ must have maximum entropy subject to the above constraint. The principle of maximum entropy is well established and has been derived axiomatically as a general principle of inductive inference [11, 12, 35]. The principle of maximum entropy determines a hypothetical probability distribution from the available partial information about a probability distribution. This hypothetical distribution which contains all the available information but no unsupported information and is unbiased and most likely to occur, is called the *unbiased reconstruction*.

Higashi, et. al. [9] developed the possibilistic counterpart of the principle of maximum entropy in order to define a suitable measure of uncertainty. This measure, called *l* uncertainty, is computed as:

$$U(f) = \frac{1}{l_f} \sum_{k=1}^{q-1} (l_{k+1} - l_k) \log_2 |c(f, l_{k+1})| \quad (2.3a)$$

or

$$U(f) = \frac{1}{l_f} \int_0^{l_f} \log_2 |c(f, l)| dl \quad (2.3b)$$

where $f = (\phi_i \mid i \in N_{|X|})$, $l_f = \max_i \phi_i$, $L_f = \{l \mid (\exists i \in N_{|X|}) (\phi_i = l) \text{ or } l = 0\}$, $\{l_1, l_2, \dots, l_q\}$, and $c(f, l) = \{i \in N_{|X|} \mid \phi_i \geq l\}$. The set L_f is called a level set of f , the function c is called the *l*-cut function and the set $c(f, l)$ is called an *l*-cut of f .

As discussed previously, U-uncertainty serves as a measure of uncertainty for possibilistic systems which satisfies some additional properties beyond those satisfied by Shannon's entropy [34] and can be used to justify the selection of a particular function from the reconstruction family in the context of possibilistic systems.

3.3 Reconstruction of Possibilistic Systems

We first define the concepts of null extension, minimal substate and partial reconstruction hypothesis. Then we describe a partial join procedure in order to compute the unbiased reconstruction from partial information.

Definition 3.1 Let β be a substate. Then $\alpha \in A$ is said to be a null extension of β if $\alpha > \beta$ and every variable of α which does not occur in β has a zero value.

Definition 3.2 Two substates are equivalent if and only if they have same null extension.

Definition 3.3 There may be more than one substate with the same null extension. The one with the least function value is said to be the minimal substate.

Definition 3.4 If the subsystems related to a reconstruction hypothesis are not complete then the hypothesis is said to be a partial reconstruction hypothesis.

It is important to note that, in the context of probabilistic reconstructability analysis, the concept of null extension is used to generate independent information. Two states are said to be equivalent if they are in the same equivalence class. It suffices to work with only independent information. We must emphasize that in the context of possibilistic reconstructability analysis, the concept of a null extension does not necessarily generate independent information. Rather, it provides a tool to carry out reconstructability analysis in the absence of complete information.

Corollary 3.1 : Let 1f and 2f be two possibilistic behavior functions such that $^1f: X_1 \times X_2 \rightarrow [0, 1]$, and $^2f: X_2 \times X_3 \rightarrow [0, 1]$. Then their join $^1f * ^2f$ is a function $^1f * ^2f: X_1 \times X_2 \times X_3 \rightarrow [0, 1]$ such that

$$[^1f * ^2f](\alpha, \beta, \gamma) = \min[^1f(\alpha, \beta), ^2f(\beta, \gamma)] . \quad (3.1)$$

Corollary 3.2: Let $f_S = *^k f$. Then f_S is unbiased reconstruction implied by S .

Corollary 3.1 and 3.2 are due to [6]. Following is the possibilistic version of the reconstruction algorithm given by Jones [15].

3.3.1 Reconstruction Algorithm

Given a consistent reconstruction hypothesis in the form of equations $\max_{\alpha > \beta} f(\alpha) = ^k f(\beta)$ for all $^k f(\beta)$ available. We obtain an unbiased reconstruction $f_{S_{\text{Partial}}}$ as follows.

- [1] $f(\alpha) := 1$ for all α in the system;
- [2] for all $^k f(\beta)$ available do
 $f(\alpha) := f(\alpha) * ^k f(\beta)$ where
 $f(\alpha) * ^k f(\beta) = \min [f(\alpha \mid \beta), ^k f(\beta)]$
 $= \min [f(\alpha), ^k f(\beta)];$
- [3] $f_{S_{\text{Partial}}} := f$; $f_{S_{\text{Partial}}}$ is an unbiased reconstruction for the available information;
- [4] stop.

We illustrate above algorithm using the following reconstruction hypothesis $\{ / v_1, v_2 / , / v_2, v_3 / , / v_1, v_3 / \}$.

v_1	v_2	^{12}f	v_2	v_3	^{23}f	v_1	v_3	^{13}f
0	0	0.8	0	0	0.8	0	0	0.8
0	1	0.5	0	1	0.7	0	1	0.7
1	0	0.0	1	0	0.8	1	0	0.8
1	1	0.8	1	1	0.5	1	1	0.5

Following are the equivalence classes for the above reconstruction hypothesis.

Equivalence Class	Null Extn
$\{ ^1(0), ^2(0), ^3(0), ^{12}(00), ^{23}(00), ^{13}(00) \}$	000
$\{ ^3(1), ^{23}(01), ^{13}(01) \}$	001
$\{ ^2(1), ^{12}(01), ^{23}(10) \}$	010
$\{ ^1(1), ^{12}(10), ^{13}(10) \}$	100
$\{ ^{13}(11) \}$	101
$\{ ^{12}(11) \}$	110
$\{ ^{23}(11) \}$	011
$\{ \}$	111

Choosing the minimal element from each equivalence class (if there are more than one

minimal elements then choosing any one of them), we get $S_{\text{Partial}} = \{ {}^{12}(00), {}^{13}(01), {}^{12}(01), {}^{23}(11), {}^{12}(10), {}^{13}(11), {}^{12}(11) \}$. The unbiased reconstruction determined by the above algorithm is $(f(000) = 0.8, f(001) = 0.7, f(010) = 0.5, f(011) = 0.5, f(100) = 0.0, f(101) = 0.0, f(110) = 0.8, f(111) = 0.5)$.

3.3.2 Proof of the Reconstruction Algorithm

We state, in view of Corollary 3.1 and 3.2, that the algorithm considers only the given information and no additional information. Thus, the algorithm computes the unbiased reconstruction for the information employed. Then $f_{S_{\text{Partial}}}$ is an unbiased reconstruction implied solely by S_{Partial} . However, we cannot say if f_S is the unbiased reconstruction for the whole system. Also, choosing minimal substate for each equivalence class does not necessarily guaranty maximum uncertainty reconstruction though it certainly provides a better resolution of the system being reconstructed. Now we prove that $f_{S_{\text{Partial}}}$ is the unbiased reconstruction solely implied by S_{Partial} .

Definition 3.5: Let $F = \{ f \mid f: A \rightarrow [0, 1] \}$. Let \leq denote a partial ordering in F such that for each pair $f_1, f_2 \in F$, $f_1 \leq f_2$, if and only if $f_1(\alpha) \leq f_2(\alpha)$ for all $\alpha \in A$.

Given a partial reconstruction hypothesis S_{Partial} we assert that the reconstruction family of S_{Partial} , $F_{S_{\text{Partial}}}$ has a unique maximum $f_{S_{\text{Partial}}}$ with respect to partial ordering \leq which can be determined by $f_{S_{\text{Partial}}} = \bigvee_k f(\beta)$ for $f(\beta)$ in S_{Partial} . Similar to Higashi, et. al. [10], let $f \in F_{S_{\text{Partial}}}$. Then $f(\alpha) \leq \min_j f(\alpha \downarrow {}^j V)$ for all α and for all $f(\alpha \downarrow {}^j V)$ in S_{Partial} . On the other hand, let there be some $\alpha_0 \in A$ such that $f(\alpha_0) > \min_j f(\alpha_0 \downarrow {}^j V)$. Then there exists some f_0 in S_{Partial} such that $f(\alpha_0) > f_0(\alpha_0 \downarrow {}^{j_0} V)$, which contradicts the very definition of projection function in possibilistic systems. Thus $f \in F_{S_{\text{Partial}}}$. Thus $f \in F_{S_{\text{Partial}}} \Rightarrow f \leq f_{S_{\text{Partial}}}$. Now, $f \leq f_{S_{\text{Partial}}} \Rightarrow \max_{\alpha > {}^j \alpha} f(\alpha) \leq \max_{\alpha > {}^j \alpha} f_{S_{\text{Partial}}}(\alpha)$ for all ${}^j \alpha$ in S_{Partial} . or

$$f({}^j \alpha) \leq f_S \downarrow {}^j V({}^j \alpha) \quad (3.2)$$

for all ${}^j \alpha \in {}^j A$ in S_{Partial} .

For some $\alpha \in A$ such that $\alpha > {}^j \alpha$, $f_S(\alpha) = \min_i f({}^i \alpha) \leq f({}^j \alpha)$ for all ${}^i \alpha \in {}^i A$ in S_{Partial} . So $f_S(\alpha) \leq f({}^j \alpha)$. But $[f_S \downarrow {}^j V]({}^j \alpha) = \max_{\alpha > {}^j \alpha} f_S(\alpha) \leq f_S(\alpha) \leq f({}^j \alpha)$.

Therefore,

$$[f_{S_{\text{Partial}}} \downarrow {}^j V]({}^j \alpha) \leq f({}^j \alpha). \quad (3.3)$$

From (3.2) and (3.3), $[f_{S_{\text{Partial}}} \downarrow {}^j V]({}^j \alpha) = f({}^j \alpha)$ establishing $f_{S_{\text{Partial}}} \in F_{S_{\text{Partial}}}$. This concludes the proof of the following theorem.

Theorem : If $F_{S_{\text{Partial}}}$ is non-empty, then $F_{S_{\text{Partial}}}$ has a unique maximum $f_{S_{\text{Partial}}}$ with respect to partial ordering \leq which can be determined by using partial join as described by the reconstruction algorithm.

3.4 Reconstruction Families of Possibilistic Systems

A reconstruction family of a given structure system can be considered equivalent to the set of solutions of a special type of fuzzy relation equations. The solution set thus obtained is partially ordered and contains both minimal solutions and unique maximum solutions. It suffices only to identify the maximum and minimal elements in order to determine the whole reconstruction family. This was the idea used by Higashi, et. al. [10] in identifying the reconstruction family of possibilistic systems. In this section, we extend the research on the same line by providing a method for partial reconstruction hypotheses.

A possibilistic measure is a special kind of fuzzy measure which is applicable only to finite sets and some special types of infinite sets [10, 32, 36]. However, we are concerned here with finite sets only. Let S_{Partial} be a partial reconstruction hypothesis. Then all functions $f \in F_{S_{\text{Partial}}}$ can be determined by solving the set of simultaneous equations

$$\max_{\alpha > {}^j \alpha} f(\alpha) = f({}^j \alpha) \quad (4.1a)$$

for all α in the system and for all ${}^j \alpha$ in S_{Partial} , along with the constraint

$$0 \leq f(\alpha) \leq 1. \quad (4.1b)$$

Equation (4.1a) can be expressed as

$$\max_{\alpha \in A} \min(f(\alpha), \delta_{\alpha, j\alpha}) = j f(j\alpha) \quad (4.2a)$$

for all $\alpha \in A$ and $j\alpha$ in S_{partial} , where

$$\delta_{\alpha, j\alpha} = \begin{cases} 1 & \text{if } \alpha > j\alpha \\ 0 & \text{otherwise.} \end{cases} \quad (4.2b)$$

This problem can now be translated in terms of fuzzy relation equations as follows. Let p , q and r be fuzzy binary relations defined as $p: X \times Y \rightarrow [0, 1]$, $q: Y \times Z \rightarrow [0, 1]$, and $r: X \times Z \rightarrow [0, 1]$, and let " \circ " be max-min composition [7, 8, 10, 33]. Then, the general form of a fuzzy relation equation can be written as

$$p \circ q = r. \quad (4.3)$$

Thus, for all $x \in X$ and $z \in Z$,

$$p \circ q(x, z) = \sup_{y \in Y} \min(p(x, y), q(y, z)). \quad (4.4)$$

Because X , Y and Z are finite sets, functions p , q , and r can be mapped onto their respective matrices. That is, $p = (p_{ij})$, $q = (q_{jk})$ and $r = (r_{ik})$, where $p_{ij} = p(x_i, y_j)$, $q_{jk} = q(y_j, z_k)$ and $r_{ik} = r(x_i, z_k)$, and $x_i \in X$, $y_j \in Y$ and $z_k \in Z$. Now for each pair (i, k) we can write

$$r_{ik} = \max_j \min(p_{ij}, q_{jk}). \quad (4.5)$$

Now we define

$$p = (p_1, p_2, \dots, p_{|A|}), p_i = f(\alpha_i), i \in N_{|A|}, \quad (4.6a)$$

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1|P_\#|} \\ q_{21} & q_{22} & \cdots & q_{2|P_\#|} \\ \vdots & \vdots & \ddots & \vdots \\ q_{|A||1} & q_{|A||2} & \cdots & q_{|A|||P_\#|} \end{bmatrix} \quad (4.6b)$$

where

$$P_\# = \{ {}^*f(\beta_1), {}^*f(\beta_2), \dots, {}^*f(\beta_{|P_\#|}) \}, \quad (4.6c)$$

$$q_{ik} = \begin{cases} 1 & \text{if } \alpha_i > \beta_k \\ 0 & \text{otherwise} \end{cases} \quad (4.6d)$$

and

$$r = (r_1, r_2, \dots, r_{|P_\#|}). \quad (4.6e)$$

Note that *f are subsystem functions in $P_\#$. Equation (4.2a) can now be rewritten as

$$\max_{i \in N_{|A|}} \min(p_i, q_{ik}) = r_k \quad (4.7a)$$

for all $k \in N_{|P_\#|}$ which essentially is

$$p \circ Q = r. \quad (4.7b)$$

Now for the given example, we have the following representation of $p \circ Q = r$,

$$\begin{bmatrix} p_0 & p_4 & p_2 & p_1 & p_6 & p_3 & p_5 & p_7 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} = [0.8, 0.0, 0.5, 0.7, 0.8, 0.5, 0.5].$$

The system of equations given above can further be simplified by eliminating those columns from Q which correspond to 0 values in r -vector. The solution to the above set of equations is $p_0 = 0.8$, $p_1 = 0.7$, $p_4 = 0.0$, $p_5 = 0.0$, $p_6 = 0.8$, $p_7 = 0.5$, and either $p_2 = 0.5$ and $p_3 \leq 0.5$, or $p_3 = 0.5$ and $p_2 \leq 0.5$. Obviously, the reconstruction family has infinite number of elements. Only one of them is maximum represented by $p_2 = 0.5$ and $p_3 = 0.5$. Two of them are minimal elements represented by $p_2 = 0.0$ and $p_3 = 0.5$, and $p_2 = 0.5$ and $p_3 = 0.0$.

5. Concluding Remarks

The very importance of system reconstruction dwells in its ability to identifying the important components during the process of system reconstruction. Klir [25] introduced the reconstruction principle of inductive inference. The process of inductive inference can be carried out in several phases. In the first phase, an

overall constraint is derived from the available data. In the subsequent phases, superior reconstruction hypotheses can be determined for the overall system at the various refinement levels. As stated earlier, this novel principle of inductive inference, embedded in the reconstruction process, has successfully been applied to the diverse fields such as generalized rule induction, expert systems, pattern classifications and distributed sensor integration [22, 25, 36].

In this paper we have attempted to strengthen this principle by extending the realm of RA for possibilistic systems with incomplete information. We have proposed a reconstruction algorithm to this effect which computed the unbiased reconstruction for the available information. Further, we have expressed the problem of determining the reconstruction family from a partial reconstruction hypothesis in terms of max-min fuzzy relation equations. Matrix Q is in the form of a lower triangular matrix which can further be compressed by eliminating the columns corresponding to zero values in the r -vector. This greatly simplifies the computation. Details of this max-min approach are listed in Higashi, et al. [10].

6. References

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